

# A SIMPLE AND SYSTEMATIC METHOD FOR THE DESIGN OF TAPERED NONLINEAR TRANSMISSION LINES

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## ABSTRACT

In this paper, we present an original method, simple to implement, rapid and systematic for the conception of Nonlinear Transmission Lines (NLTL) for shock wave generation. This method is based on SPICE simulations and allows the synthesis of hybrid or monolithic NLTLs.

## INTRODUCTION

NLTL have been investigated since more than thirty years with the work of Landauer [1]. More recently, Jäger [2] and Rodwell [3] have demonstrated the ability of NLTL to both generate shock waves with subpicosecond falltime and to propagate ultrafast solitons. Nevertheless, to our knowledge, no systematic method for the design of NLTLs has been published at this time. Some informations are given in [4] : it appears that the choice of the number of sections is tricky and thus numerical simulations are necessary.

The focus of this paper is the problem of the conception of NLTL for shockwave generation. A systematic method is presented to determine the number of sections of the NLTL in order to obtain a desired value  $t_f$  of the shockwave falltime, when the NLTL is fed by a pulse with a  $t_i$  falltime. When the nonlinear component of the NLTL is a diode, our method allows one to calculate the necessary voltage variable capacitance of the diode.

The paper is organized as follows: first, the principles of NLTLs are recalled. The problem of metallic losses is quickly addressed to show the relevance of tapered NLTLs. Then the method and the hypotheses are described.

Finally this method is applied to the case of a GaAs monolithic NLTL.

## NLTL PRINCIPLES

A NLTL (fig. 1a) consists of Coplanar Wave guide (CPW) transmission lines of impedance  $Z_l$ , periodically loaded by reverse-biased Schottky contacts, which serve as voltage-variable capacitors. If  $v_{CPW}$  is the propagation velocity of the unloaded CPW and  $d$  the distance between two diodes, we can write the delay time as :  $\tau = d / v_{CPW}$  (1).

Fig. 1b shows the equivalent circuit, where  $L_l = Z_l \tau$  (2) and  $C_l = \tau / Z_l$  (3) are the CPW section inductance and capacitance.  $C_d(V)$  and  $R_d$  are respectively the voltage-variable diode capacitance and series resistance.

The large signal impedance of the NLTL is given

$$\text{by: } Z_{ls} = \sqrt{\frac{L_l}{C_l + C_{ls}}} \quad (4) \text{ with: } C_{ls} = \frac{\int_{V_l}^{V_h} C_d(V) dV}{V_h - V_l}$$

where  $V_h$  and  $V_l$  are given on Fig. 1a.

$C_d(V) = C_{j0} \cdot f(V)$  where  $C_{j0}$  is the zero-biased diode capacitance value and the expression of  $f(V)$  depends on the doping profile of the diode.

The Bragg cutoff frequency  $f_B = \frac{1}{\pi \sqrt{L_l (C_l + C_{ls})}}$

(6) and the diode capacitance large signal cutoff frequency  $f_{c,ls} = \frac{1}{2\pi R_d C_{ls}}$  (7) are the time-compression limitation factors. The minimum falltime that one can obtain for a given NLTL depends on  $f_B$  and  $f_{c,ls}$  [3].

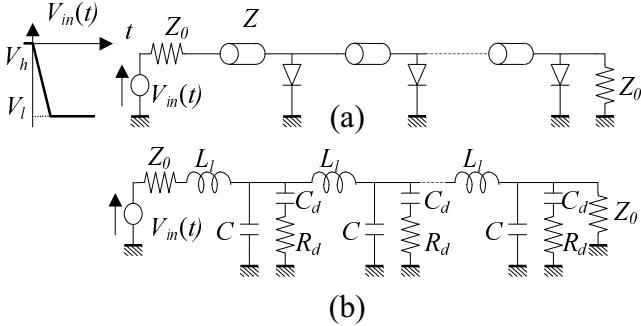


Figure 1. (a) Circuit diagram with input signal  $V_{in}(t)$  and (b) equivalent circuit of NLTL.

### METALLIC LOSSES

If a monolithic fabrication is considered with conductors thickness of the order of  $\mu\text{m}$ , CPW metallic losses can not be neglected, more especially as the frequency is higher than a few tens of GHz. The decrease of the conductor thickness leads to an increase of the DC resistance  $R_{dc}$  and the increase of the frequency makes the skin effect critical. The total series resistance of a transmission line can be approximated as :  $R = R_{dc} + k\sqrt{\omega}$  (8) where the term  $k\sqrt{\omega}$  corresponds to the skin effect.

$R_{dc}$  reduces the amplitude of the pulse propagating along the NLTL. If we consider only this term, for low losses conditions corresponding to  $R_{dc} \ll L_l \omega$  (which is generally the case for NLTLs), and if we consider the case  $t_r \ll \pi\sqrt{L_l(C_l + C_{ls})}$ , the pulse amplitude at a distance  $x$  from the input can be written as [5]:

$$V\left(x, t + \frac{x}{d} \cdot \sqrt{L_l(C_l + C_{ls})}\right) \quad (9)$$

$$\approx V_{in}(t) \exp(-R_{dc} \cdot x / 2Z_{ls})$$

Simulation results show that the attenuation due to  $R_{dc}$  can be critical.

The term  $k\sqrt{\omega}$  introduces temporal-distortion because the attenuation increases with frequency. This attenuation competes with the nonlinear behavior of the NLTL, which can be understood as a reshaping of the input pulse by a

strengthening of high frequencies (see figure 5 at the end of this paper).

As a consequence, skin effect decreases the compression efficiency obtained with the diodes variable capacitance.

In practice, skin effect can be simulated with SPICE in the Time Domain by using a convolution technique [6]. In the case of NLTLs, this solution is not suitable with PC-like computers because the necessary simulation time is greater than a few months. This is due to a very small calculation temporal increment together with the large number of sections with small lengths.

### ORIGINAL CONCEPTION METHOD

The final falltime  $t_{r,out}$  (10%-90%) that one can obtain at the output of a given NLTL is fixed by the Bragg frequency  $f_B$  and the diode cutoff frequency  $f_{c,ls}$ . In practice, Rodwell has shown in [3] that  $f_B$  and  $f_{c,ls}$  should be related by:  $4 \leq \frac{f_{c,ls}}{f_B} \leq 6$  (10). This

choice results from a compromise between the overshoot amplitude at the end of the NLTL and the NLTL's length. In these conditions,  $t_{r,out}$  is mainly imposed by  $f_B$ . So the determination of  $f_B$  can be done directly from  $t_{r,out}$ . For an input pulse with a falltime  $t_r$ , the time-frequency product can be expressed as:  $\omega_e \cdot t_r = K$  (11) where  $2 < K < 3$ ,  $\omega_e$  is the cutoff frequency of the energy spectrum of the pulse. For  $K=3$ , we obtain the maximum value  $\omega_{emax}$  of  $\omega_e$ . Therefore, taking  $\omega_B = \omega_{emax}$  ensures that the NLTL will propagate all the frequency components of the time-compressed signal. Thus, for the NLTL modeling, we use:

$$f_B = \frac{\omega_B}{2\pi} \approx \frac{0.48}{t_{r,out}} \quad (12)$$

Then one calculates  $C_{ls}$  for a given couple of  $Z_l$  and  $Z_{ls}$ . Since the propagation velocity of the NLTL (loaded CPW) can be written as  $V_{CPW,ls} = d / \sqrt{L_l(C_l + C_{ls})}$ , and using relations (1)-(3) and (6), we obtain:  $C_{ls} = \frac{(1 - Z_{ls}^2 / Z_l^2)}{\pi Z_{ls} f_B}$  (13).

As shown in [3],  $Z_l$  must be taken between 70 and 80  $\Omega$ , this value resulting from a tradeoff between metallic losses and the NLTL's length (the relation between  $Z_l$  and metallic losses results from layout considerations).

The knowledge of the amplitude of the input pulse gives  $C_{j0}$  from  $C_{ls}$  by using equation (5).

The final step of this method consists in determining the number  $N$  of sections of the NLTL. This is made by the mean of abacus derived from SPICE simulations, and gives  $t_{r,out}$  in function of  $f_B$  with  $N$  as a parameter. When the Bragg frequency is increased, one obtains an optimum  $f_B$  value giving the best falltime. Above the optimum frequency, the falltime takes more longer values because  $C_{ls}$  decreases as the Bragg frequency increases (relation (13)): therefore the pulse compression is not fully achieved. As the number of sections of the NLTL is large and several simulations have to be done to draw the abacus, we have implemented a code that automatically assigns the electrical parameters to each section and runs all necessary simulations. This procedure is valid for tapered NLTLs with  $f_{B,in} \neq f_{B,out}$ .

Using the same criteria as previously (relation (12)), we have:  $f_{B,in} > \frac{0.48}{t_{r,in}}$  (14).

## EXAMPLES

The characteristics of a monolithic NLTL are given in table 1. A 1.6 ps falltime is desired. The input pulse falltime (10%-90%) is 16 ps with  $V_h = 0$  and  $V_l = -12$  V.

$t_{r,out}$	$Z_l$	$Z_{ls}$
1.6 ps	70 $\Omega$	50 $\Omega$

Table 1. NLTL characteristics.

From  $t_{r,out}$  is derived  $f_B = 300$  GHz. Relations (13) and (4) give  $C_{ls} \approx 10.4$  fF and  $C_{j0} \approx 21.3$  fF for a uniform Schottky diode doping profile. From the abacus of figure 2,  $N \approx 90$  is deduced.

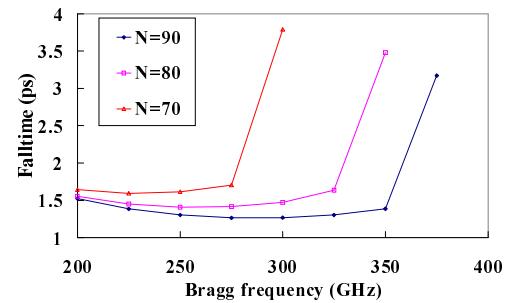


Figure 2. Abacus for the determination of the number of section  $N$  for the NLTL defined in table 1.

For  $f_B = 300$  GHz, the minimum output falltime is equal to about 1.3 ps for  $N=90$ . This value is close to the expected falltime (1.6 ps).

The simulated temporal shapes of the input and compressed pulses are given in figure 3.

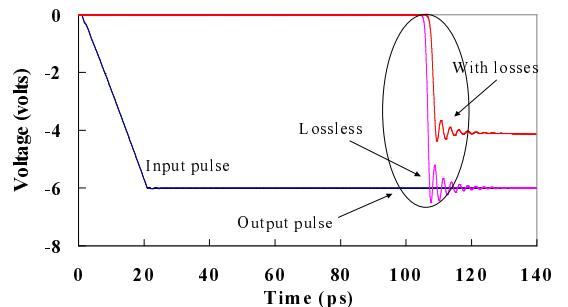


Figure 3. Simulation result for the NLTL defined in Table 1. The lossy case corresponds to  $R_{dc} = 0.45 \Omega$  per section for a CPW conductor with a thickness of 0.5  $\mu\text{m}$  and  $W = 10.2 \mu\text{m}$  ( $\sigma = 37.10^6 \text{ S/m}$ ).

For a tapered NLTL,  $f_{B,in}$  equals 30 GHz and we find  $N=39$ . We have also performed simulations with  $f_{B,in} = 100$  GHz ( $N=47$ ) and  $f_{B,in} = 200$  GHz ( $N=70$ ). Results are given in figure 4. Each of these couples ( $N, f_{B,in}$ ) gives the same output falltime  $t_{r,out} \approx 1.6$  ps.

The length of each NLTL and the pulse amplitude of the output pulse (from fig. 4) are given in table 2.

Tapered ( $f_{B,in}=30$ GHz)	Tapered ( $f_{B,in}=100$ GHz)	Tapered ( $f_{B,in}=200$ GHz)	Uniform ( $f_B=300$ GHz)
8.8 mm	7.3 mm	7.4 mm	7.7 mm
-5.0 V	-4.9 V	-4.5 V	-4.1 V

Table 2. NLTL lengths with output pulse amplitudes.

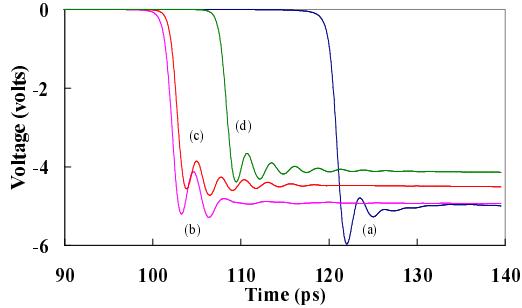


Figure 4. Simulation results for (a) tapered NLTL with  $f_{B,in} = 30$  GHz, (b) tapered NLTL with  $f_{B,in} = 100$  GHz, (c) tapered NLTL with  $f_{B,in} = 200$  GHz, (d) uniform lossy NLTL.

The pulse output amplitude decreases when  $f_{B,in}$  increases. This justifies the use of tapered transmission lines. Moreover this effect will be reinforced in practice due to skin losses. The length of the NLTL decreases between the uniform NLTL and the tapered NLTL with  $f_{B,in} = 100$  GHz. For the tapered NLTL with  $f_{B,in} = 30$  GHz, the length is increased. This is due to the length of the first sections. So, for the case considered here, the best choice is to take the tapered NLTL with  $f_{B,in} = 100$  GHz because the pulse amplitude do not significantly differs from the NLTL with  $f_{B,in} = 30$  GHz. However, the design of higher frequency NLTLs shows that losses effects become more and more critical and the choice of the smaller input Bragg frequency becomes the best choice.

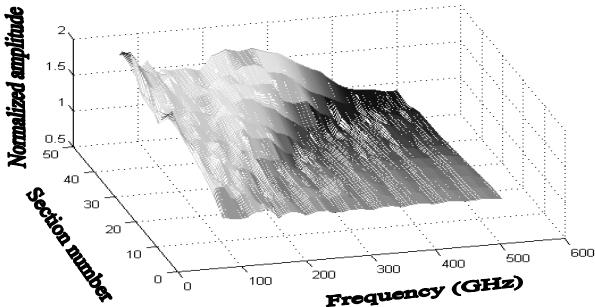


Figure 5. Normalized spectrum envelope.

Finally, figure 5 shows the normalized spectrum envelop of the pulse along the NLTL in the case of a tapered NLTL with  $f_{B,in} = 100$  GHz. It appears clearly that high frequency components are strengthened.

## CONCLUSION

We have demonstrated a simple and systematic method for the synthesis of NLTLs. At the conference, effects of the change of  $Z_l$  will be discussed. We will also address the problem linked to layout constraints, especially the transmission line cross-section constraint.

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